A SIMPLE NOTCH FILTER FOR REMOVING HIGH FREQUENCY NOISE IN ATMOSPHERIC MEASUREMENTS

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The experimenter occasionally encounters high-frequency (> 1 Hz) contamination in sonic anemometer data that adversely affects higher-order moment calculations. One has the option of applying a low-pass filter (either a simple, equally weighted moving average filter or any one of the sharper cut-off versions available today) to eliminate not only the unwanted frequency, but all frequencies above it as well. A band rejection or notch filter would be a better choice because of its selectivity, but is seldom used because of computational complexity. In many instances the contamination is quasi-sinusoidal, often caused by vibration of the supporting boom at a frequency typically between 1 and 2 Hz. A simple procedure exists for removing such noise without seriously degrading the signal. It is a notch filter, realized by averaging data points spaced $\tau/2$ apart in time, where τ is the period of the noise signal.

$$y_i = \left(x_i + x_{i+\tau/2\Delta t}\right)/2 \quad ; \quad i = 1, 2, 3, \ldots, N \tag{1}$$

x_i represents the contaminated time series and y_i the new filtered time series; Δt is the interval between the successive samples and we assume for convenience that $\tau/2\Delta t$ is an integer.

To derive the transfer function for this process we will regard the signal as being continuous in time; x(t) and y(t) will replace time series x_i and y_i and their Fourier transforms in the frequency domain become X(f) and Y(f).

$$Y(f) = \int_{-\infty}^{\infty} e^{i2\pi ft}\, y(t)\, dt \tag{2}$$

$$= 1/2\left[X(f) + e^{-i\pi f\tau} \cdot X(f)\right] \tag{3}$$
$$= X(f) \cdot 1/2\left(1 + e^{-i\pi f\tau}\right)$$

The power spectrum $S_y(f)\left[= |Y(f)|^2\right]$ becomes

$$S_y(f) = S_x(f) \cdot 1/2\left(1 + \cos \pi f\tau\right) \tag{4}$$

$$= S_x(f) \cos^2\left(\pi f\tau/2\right)$$

The transfer function K(f) for the process will be

$$K(f) = S_y(f)/S(f) = \cos^2 \pi f\tau/2 \tag{5}$$

This function, shown in Figure 1, has zeroes at $f = n/\tau$, where n is an odd number. For comparison, the transfer function $\sin^2\pi f\tau/(\pi f\tau)^2$ for an equally weighted moving average filter of width τ is also shown on the same plot. This function drops to zero at $f = 1/\tau$, but slightly more rapidly than the cosine-squared function, and its lobes diminish in amplitude with increasing frequency, while those of the latter do not. By averaging data points spaced

$\tau/2$ apart in the time series we are, in fact, averaging pairs of points that are 180° out of phase. With the 10 Hz sampling rate in the ATI sonic anemometer, one can place the first notch at 5, 2.5, 1.67, 1.25, or 1 Hz (by averaging points separated by 1, 2, 3, or 5 sampling intervals apart, respectively, depending on which of the frequencies falls closest to the unwanted frequency in the signal.

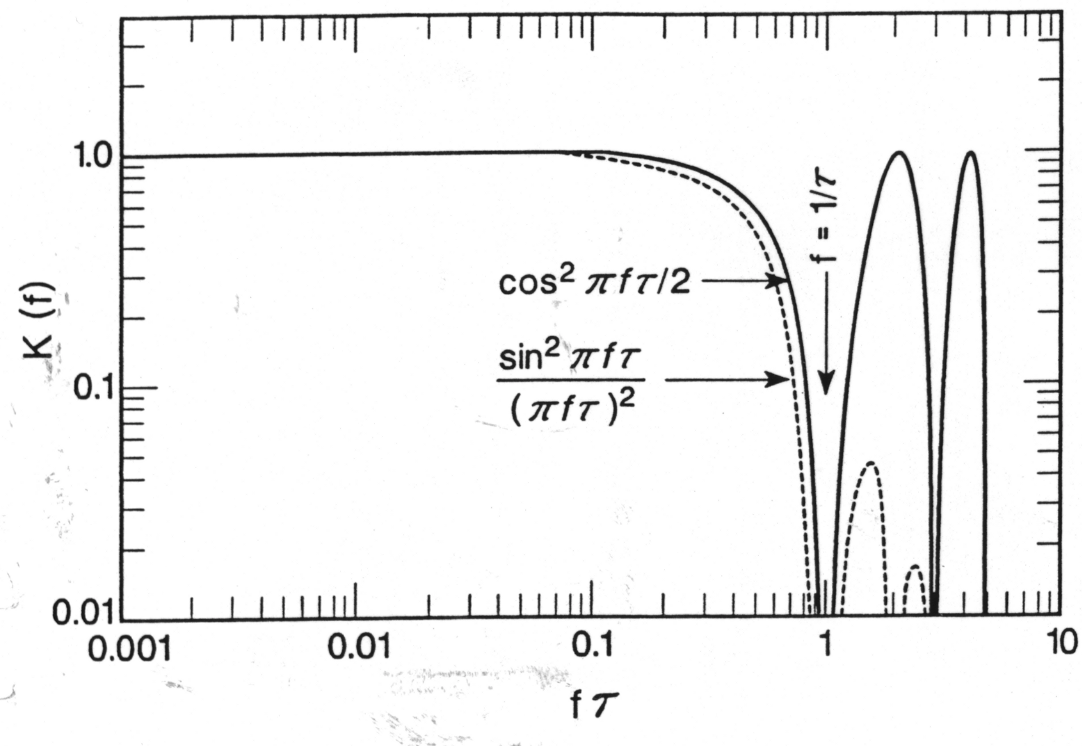

Figure 1. Transfer function for the cosine squared notch filter compared to that for a moving average rectangular filter.