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MOMENTUM FLUX: GROSS, SCALAR, ALONGWIND, OR NET?

Introduction.

Modern micrometeorology is an amalgam of two traditions: one arises from field measurements in the open atmosphere and another from wind tunnel experiments. Early micrometeorological field work, and even relatively recent studies (viz. the von Karman constant experiments by Frenzen and Vogel, 1995), were done with precision cup anemometers, which can only provide a scalar speed measurement. Most of the early flux/gradient measurement programs (see Dyer, 1974) used scalar wind measurements, and derived quantities such as friction velocity (u_*), logarithmic wind profiles, and Monin-Obukhov Similarity Theory (MOST) relationships based on these measurements. Businger et al. (1971) made a distinction between the mean horizontal (scalar) wind speed and the magnitude of the mean horizontal wind vector. However, Businger (1973) reverted to the old habit of referring to the mean (scalar) horizontal wind speed $\underline{u}$ (the underbar signifies an averaged or mean quantity) in his work on turbulent transfer in the atmospheric surface layer. Meanwhile, velocity component and Reynolds stress experiments done in wind tunnels relied primarily on hot-wire anemometry (see, for example, Bradshaw, 1971), which provided orthogonal velocity component measurements. With the advent of sonic anemometry (sonics), atmospheric measurements of alongwind u, crosswind v, and vertical w velocity components have become routine. In consequence, eddy correlation techniques taken from the laboratory tradition are used in computation of a "vector" momentum flux F_n and derived quantities such as friction velocity u_* and Obukhov length L. Other flux computations are also possible, including the scalar, gross, and alongwind flux. Given the capacity to compute momentum flux in a variety of different ways, which way is most appropriate for MOST applications?

1. Flux Computation Alternatives.

The flux of a scalar quantity is simply a mean of the individual products of w with that generalized scalar quantity a:

$$F_a = S(w_i a_i)/n = \underline{w'a'} - \underline{w}\ \underline{a} \qquad (1)$$

where the subscript i signifies the ith measurement, S indicates summation from the first to the nth measurement, and primed quantities represent departures of individual measurements from a time-averaged mean. (The means of individual primed quantities are zero and the mean vertical velocity $\underline{w}$ is assumed to be zero). Given u and v component measurements, one can invoke a trigonometric identity for scalar wind speed $s = (u^2 + v^2)^{1/2}$ = usin? + vcos? , where ? is the (double argument) arctangent of u and v. When used in a product with w, this produces a scalar flux

$$F_s = S[w_i(u_i^2 + v_i^2)^{1/2}]/n = S[w_i(u_i \sin?_i + v_i \cos?_i)]/n = S(w_i s_i)/n = \underline{w's'}. \qquad (2)$$

The situation with “vector” momentum flux is more complex, and can produce disparate results depending on how the computation is performed. MOST relationships found in some references (viz. Stull, 1984) present F_n and related quantities as products of the square root of the sum of the squares of the averaged product of w with u and v:

$$F_n = -[(Sw_iu_i/n)^2 + (Sw_iv_i/n)^2]^{1/2} = -[(\underline{w'u'})^2 + (\underline{w'v'})^2]^{1/2}, \qquad (3a)$$

which can also be presented as products with sines and cosines,

$$F_n = -[(Sw_iu_i/n)\sin\underline{?} + (Sw_iv_i/n)\cos\underline{?}] = -[\underline{w'u'}\sin\underline{?} + \underline{w'v'}\cos\underline{?}], \qquad (3b)$$

where $\underline{?}$ is determined by a double argument $\tan^{-1}(\underline{w'u'}/\underline{w'v'})$. Friction velocity and L are given by

$$u_* = [(\underline{w'u'})^2 + (\underline{w'v'})^2]^{1/4}, \qquad (4)$$

$$L = [(\underline{w'u'})^2 + (\underline{w'v'})^2]^{3/4}/[k(g/\underline{T}_s)(\underline{w'T_s'})], \qquad (5)$$

where T_s is the sonic-derived temperature (ºK), and $\underline{w'T_s'}$ is the (scalar) temperature flux. This quantity is described as a “net” flux for reasons described below.

The gross flux F_g is the mean of the absolute products of w with u and v components

$$F_g = S[|w_i(u_i^2 + v_i^2)^{1/2}|]/n, \qquad (6)$$

and the alongwind flux F_a is the flux of the alongwind component of the wind after a software (two coordinate) rotation into the mean horizontal wind, with $\underline{v} = 0$

$$F_a = S(w_iu_{ia})/n \qquad (7)$$

2. Evaluating Momentum Flux Computation Method Differences.

The scalar momentum flux is easy to understand because it is the mean of the sum of the individual products of w with s. The sign of F_s is determined by the cumulative sum of the individual products of ±w with its corresponding positive scalar speed, or equivalently with ±s´, and as such is computed in the same way that the flux of any other scalar quantity is computed.

Net momentum flux arises from the square root of the summed squares of the means of the products of w with u and with v. The resulting equations are problematic in that, while the mean of the sums of the products of w with u or v can be of either sign, this sign is lost by taking the square root of the sum of the squares (Eqn. 3a) or by applying its trigonometric equivalent (Eqn. 3b). Thus, the sign of F_n depends on whether or not the user applies a negative sign to the equation rather than on the direction of net momentum flux. Likewise, Eqns. (4) and (5) do not take the sign of the flux into account. In the laboratory, where the wall is the momentum sink and momentum flux is always directed towards the wall, Eqns. (3a) and (3b) can be used without a problem. However, the atmosphere offers far more complexity; $\underline{w'u'}$ and $\underline{w'v'}$ can take either sign. Friction velocity and L cannot be properly defined if momentum flux is positive, even though these equations will produce what appears to be a mathematically correct result.

Gross flux differs from the scalar flux in that the direction of the flux is lost with computation of absolute values of each product of w with s. As with the net flux, the sign or direction of the flux is lost. Gross flux can be thought of as the sum of a correlated component that appears in other flux computation procedures, plus a random or uncorrelated component. It is therefore more an indicator of turbulence than of flux. Biltoft (2000) shows that F_g is directly proportional to and highly correlated with vertical velocity variance.

Alongwind flux, computed using only the alongwind component of the wind speed, is most appropriately considered a “vector” flux because it retains its directional component through its computation. Computation of F_a is performed after rotation into the mean wind direction. The product of w with the rotated crosswind component is not used in the alongwind flux; the sign of the crossing wind component is arbitrarily determined by whether crosswind from the left or right is chosen to be positive.

Distinct relationships arise from the computation of the different fluxes. Gross flux is the greatest in absolute magnitude, and F_a, because it lacks a contribution from the crosswind component, is likely to be the smallest in absolute magnitude. Scalar flux lies between these two: $F_g = |F_s| = |F_a|$?Scalar flux can approach F_g only in the unlikely event that w does not change sign during the measurement period (thereby violating $\underline{w} = 0$). Likewise, F_a approaches the magnitude of F_s only in the unlikely event that crosswinds are zero and changes in speed are due solely to alongwind pulses or gusts. The relationship with F_n is more complicated. For any given data set, F_s and F_g remain constant so long as the magnitudes of the u and v components do not change, while F_n can assume values greater than or less than F_s depending on the sums of products of w with u and v.

It is useful to consider the behavior of the various forms of momentum flux as they approach limits. Considering turbulence as the sum of correlated plus uncorrelated motions, all fluxes must approach zero as turbulence diminishes towards zero**.** If flux exists only when turbulence is present, does it also exist when the mean flow is zero and only turbulence is present? Alternatively, is a sustained horizontal wind direction a requirement for momentum exchange to occur? Conceptually, there is no reason why momentum flux cannot occur even if $\underline{u} = \underline{v} = \underline{w} = 0$, as long as some correlation between instantaneous horizontal and vertical components exists. The atmosphere approaches this condition at the free convection limit, or during a nocturnal calm. Measured over finite time periods during very low wind speeds, F_a approaches zero, but varies between small values of either sign. Likewise, the $\underline{w'u'}$ and $\underline{w'v'}$ components of F_n can assume either sign, rendering it an unreliable flux indicator in light winds. Because the instantaneous horizontal wind component can only be zero or a positive quantity, F_s and F_g exhibit the least random behavior, and F_s provides the true momentum flux direction. Because it does not distinguish between correlated and uncorrelated turbulent components, F_g is more indicative of turbulence than of a true flux. Therefore, only F_s and F_a can provide a true representation of flux in light and variable winds.

3. What is MOST Consistent?

As mentioned in the introduction, MOST is an amalgam of "scalar" and "vector" traditions. This produces certain inconsistencies that should be sorted out. For example, Mahrt et al. (2001) argue that defining drag coefficient C_D as a ratio of momentum flux to the time-average of the instantaneous (scalar) wind speed is inconsistent because: "(1) the drag coefficient is computed from a vector average of the stress and scalar average of the wind speed. (2) similarity theory requires the vector-averaged wind and (3) the time-average of the instantaneous wind includes contributions from the turbulence." It is true that taking C_D as the ratio of a scalar wind to a vector flux introduces an undesirable inconsistency. Perhaps using the ratio of the scalar wind to the scalar flux would be more appropriate. The MOST requirement for a vector-averaged wind is questionable, considering that much of its development occurred using scalar measurements.

If it is inconsistent to use vector and scalar averaged quantities for C_D, then the use of scalar and vector ratios for MOST variables should be reconsidered. Obukhov length, for example, could be computed using scalar momentum flux in ratio with the scalar temperature flux $(\underline{w'T'})/(\underline{w's'})^{3/2}$. This would make L more consistent with scalar-averaged winds used with the logarithmic wind profile equation and with relationships used to determine the von Karman constant.

4. Conclusions.

This note is intended to initiate discussion of scalar vs. momentum flux, rather than be the last word on this subject. The commonly used net (or vector) flux is problematic because the sign of $\underline{w'u'}$ and $\underline{w'v'}$ are lost in the computation process. This becomes a particularly significant issue for flux computation in light winds. Because it is more consistent when used in a ratio with other scalar fluxes, and retains the true flux direction with the least erratic results in low winds, the scalar momentum flux appears to be the eddy correlation momentum flux measurement of choice for MOST applications.

5. References.

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